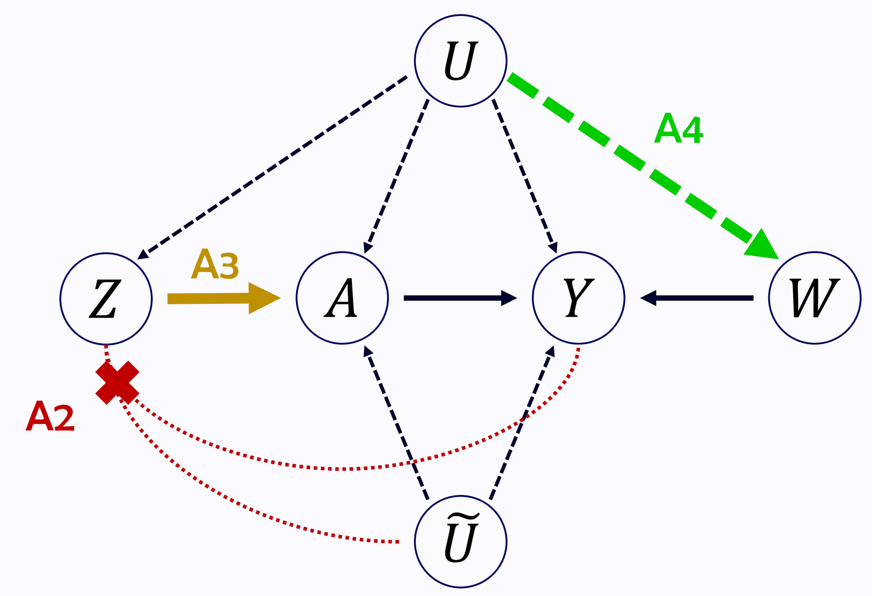
Theory

Problem Setup

Y Outcome	Z Instrument	U Common Confounder
A Treatment/Action	W Negative Control Outcome	$\tilde{U}$ Non-Common Confounder

A-ICC Simple Common Confounding Model

1. Consistency: Y=$Y(A, Z)$ and W=$W(A, Z)$
2. Conditional instrument exogeneity: $(Y, W) \perp\!\!\!\perp Z \mid (A, U)$
3. Instrument relevance (completeness): For any $g(A, U) \in L_2(A, U)$, $E[g(A, U)|Z] = 0$ only when $g(A, U) = 0$.
4. Negative control outcomes: $W \perp\!\!\!\perp (A, Z) \mid U$.
   There exists some function $h_0(A, U) \in L_2(A, U)$ such that $E[h_0(A, W) \mid A, U] = E[Y|A, U]$ a.s.



Causal effect of interest: $J = E\left[\int_{\mathcal{A}} Y(a)\pi(a)d\,\mu_A(a)\right]$ (e.g. ATE of A on Y with U conditioned out)

A-LS Linearly Separable Outcome Model: $Y = k_0(A, U) + \epsilon_Y$ with $E[\epsilon_Y|Z, U] = 0$.

Bridge Function Identification

Outcome bridge functions: $H_0^{out} = \{h \in L_2(A, W): E[k_0(A, U) - h(A, W)|A, U] = 0\} \neq \emptyset$

Observable bridge functions: $H_0^{obs} = \{h \in L_2(A, W): E[Y - h(A, W)|Z] = 0\} \neq \emptyset$

L4.0.2 Under A-LS, A-ICC, any $h_0 \in H_0^{out}$ satisfies that $E[Y - h_0(A, W)|Z] = 0$.

L4.0.3 Under A-ICC, $H_0^{out} = H_0^{obs}$.

T4.1 Let $\mathcal{T}: L_2(A, W) \to L_2(W)$: $(\mathcal{T}h)(w) = \int_{\mathcal{A}} h(a, w)\pi(a)d\,\mu_A(a)$ (linear operator).
With A4.1 and A3.1, $\forall\, h_0 \in H_0^{obs}$, $J = E[\widetilde{\phi_{IV}}(W; h_0)]$, where $\widetilde{\phi_{IV}}(w; h_0) = (\mathcal{T}h_0)(w)$.

- Identification is also possible with first stage strict monotonicity and additional common confounding richness requirements (A5.1 and A5.2 in Tien [2022]) instead of A-LS

Related Identification Approaches

Assumptions …	Instrumental Variables	Negative Control	Mixture Models
stronger in ICC	+ A3: Z must be relevant for A **conditional on U** + A4: W must be available	+ A3: Z must be **relevant for A** conditional on U	+ Want to identify at most ATE, not entire model
weaker in ICC	- A3: Z can be exogenous **conditional on U**	- A2: **A can be endogenous** conditional on U (only Z must be cond. exogenous)	- A2, A4: Only need 2, not 3, independent variable sets conditional on U

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Instrumented Common Confounding

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Application: Returns to Education

Problem Setup

- Common confounders U prevalent in economics: Ability [Griliches, 1977], attitude [Green et al., 1998]
- Treatment result of individuals' choice with heterogeneous returns (endogeneity) [Heckman et al., 2006]
- Endogenous or weak instruments often used [Cunha et al., 2006; Carneiro & Heckman, 2002]

Y Household net worth at 35	Z Pre-college GPA measures	U Ability
A BA degree	W Health measures at 29	$\tilde{U}$ Other biases (selection)
X Individual/family characteristics (sex, parental education/net worth, etc)		*Notes: Data from NLS97.* $n = 3{,}708$.

A-LL Local Linear Common Confounding Model

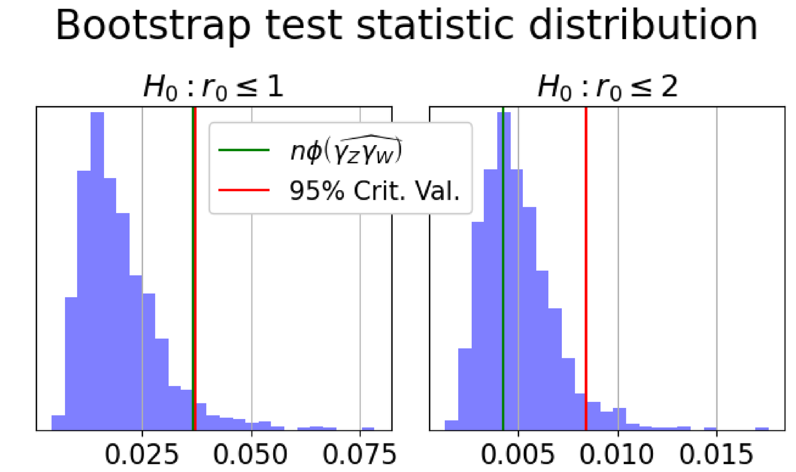
1. Model linearity: $Y = \alpha_Y(X) + A\beta(X) + U\gamma_Y(X) + W\zeta(X) + \epsilon_Y$, $W = \alpha_W(X) + U\gamma_W + \epsilon_W$,
   $A = \alpha_A(X) + Z\zeta(X) + U\gamma_A(X) + \epsilon_A$, $Z = \alpha_Z(X) + U\gamma_Z + \epsilon_U$.
2. Conditional instrument exogeneity: $E[\epsilon_Y|Z, U, X] = 0$.
3. Instrument relevance (completeness): $\text{rank}(\zeta(x)) \geq d_A$ for any $x \in \mathcal{X}$ and $\text{rank}(\gamma_Z) \geq d_U$.
4. Negative control outcomes: $E[\epsilon_W | Z, U, X] = 0$ and $\text{rank}(\gamma_W) \geq d_U$.

Causal effect of interest: $\beta(X)$ (ATE of A on Y with U conditioned out)

Test Relevance of Z and W for U

- Test $H_0: r_0 \leq r$ vs $H_1: r_0 > r$ (where $r_0 = \text{rank}(\gamma_Z\gamma_W)$ [Chen & Fang, 2019]).
- Conclusion: Likely $d_U = 1$, max. 2.
- A3 [$\text{rank}(\gamma_Z) \geq d_U$] and A4 hold.

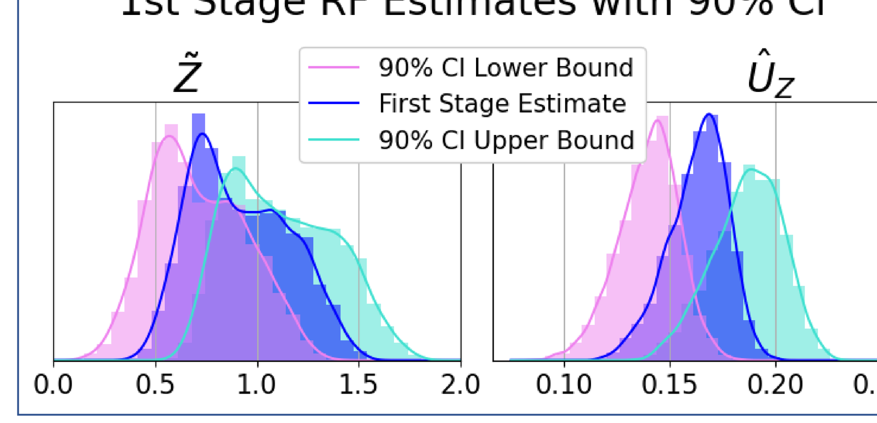
Bootstrap test statistic distribution



Test Relevance of Z for A (given U)

- Random forest for first stage with
  - $A - \hat{A}_X$ (outcome), X (covariates)
  - $\tilde{Z} - \hat{\tilde{Z}}_X$ and $\hat{U}_Z - \widehat{\hat{U}}_Z$ (LL regressor)
- A3 holds: $\text{rank}(\zeta(x)) \geq d_A$ for any $x \in \mathcal{X}$.
- $\hat{U}_Z$ positively affect A

1st Stage RF Estimates with 90% CI

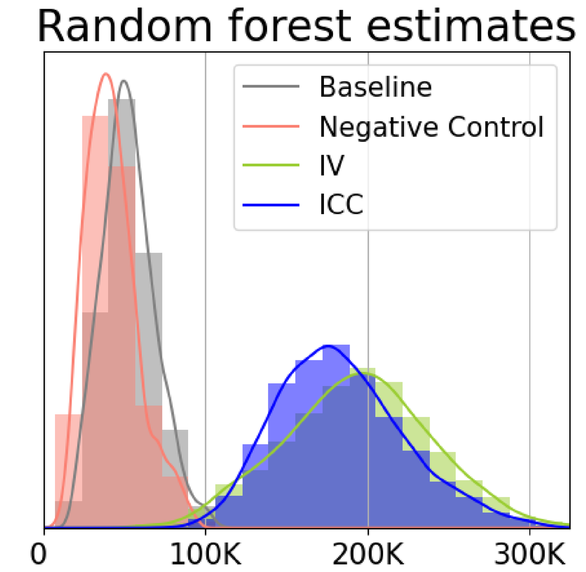


Theoretical Argument for Conditional Exogeneity of Z (given (U, X))

- Pre-college scores Z reflect ability U. Ability U affects socioeconomic and health outcomes W [Gottfredson, 2004].
- The effect is not through education A [Farrell & Fuchs, 1982]. So, ability U correlates Z and W.
- Pre-college scores Z don't affect net worth Y directly, Z only explain education A and correlate with ability U.
- Other background characteristics may matter [Link et al, 2008], so we include those in X.

Results

Random forest estimates



- IV and ICC estimates much larger than Baseline and Negative Control
- × Ability U causes little positive bias
- ICC close to IV estimates and for nearly everyone significantly positive
- ✓ Confirm earlier IV results [Card, 2001] with different approach/instruments
- New evidence for important problem

ICC RF Estimates with 90% CI